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A 3-E QUANTITATIVE DECISION MODEL OF TOXIC SUBSTANCE CONTROL THROUGH CONTROL TECHNOLOGY USE IN THE INDUSTRIAL ENVIRONMENT

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Summary

This paper presents a quantitative model which combines principles of engineering, economics, and epidemiology to illustrate a possible decision outcome from the complex interplay of various interest groups to control toxic substances in the workplace. An analysis is made of the economic motivation of the Firm, and the axiological expectations of society. This model minimizes total cost for society. A numerical sensitivity analysis is performed on two of the model parameters. A mathematical Decision Index is developed and certain conclusions drawn on the value dichotomies that are involved during the fashioning of industrial health policy.

Introduction

We as a society are the beneficiaries of technology. However, technology has affected certain groups of people more adversely than others. In the industrial environment where toxic chemicals are used to produce the goods that apparently add to our quality of life, the same processes can have a detrimental or even fatal impact on the health of the worker. The world is neither risk-free nor resource infinite. How are the inherent risks of industrial toxic chemicals be allocated amongst members of society, who as a whole, enjoy the benefits? Questions to be answered include (1) how much of society's resources should be spent to alleviate the risks of toxic substances, and (2) who should bear the costs of the control measures.

This paper gives a "3-E" mathematical model that integrates principles of engineering, economics and epidemiology to illustrate some of the subtleties of benefit maximization of toxic substance control in the industrial environment. The model recognizes total cost to society and this cost includes costs to workers, industry, and consumers. Hopefully, the model also helps illuminate the starting points or possible paths, albeit in a simplified manner, to solutions of this pressing problem.

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Definition of the model boundary

For ease of illustration, the model is built on certain simplifying assumptions. The following entities fall within the model boundary. *Society* is the entity that encompasses all the other member entities. Of the member entities, the *Firm* is a corporate entity which produces Miragood, a chemical product bought by the third entity, *Consumer*. A small fraction of the Consumer is employed by the Firm as our last entity, *Worker*. In reality there are other varieties of firms, workers, and sundry entities in Society, but for developing the model, their effects will be ignored and they will be treated simply as Consumer. The time boundary for the model is the short run which is defined below. Each entity is described in detail next.

The Firm

The Firm is a corporate entity organized to efficiently produce the chemical Miragood from raw material NOX. The existence of the Firm depends on its ability to produce Miragood within cost constraints as measured by its business accounting system. The selling price of Miragood depends on the demand and supply curve for Miragood. In the long run, the Firm will produce Miragood at a marginal cost that is less than or equal to the marginal revenue from the sale of Miragood. Marginal cost is the cost for the last additional unit produced. Total cost is the sum of the fixed and variable costs.

Analytically, the profit of the Firm is maximized when the marginal cost is equal to the marginal revenue. As the firm is the only producer of Miragood, the behavior described above is, of course, that of a monopolistic producer. This special assumption of monopoly is held throughout the first part of the analysis, but a discussion of the ramifications of control technology under more competitive market conditions will be discussed after development of the central ideas. In this model, short run is defined as the time period during which the total production costs before installation of control technology for the Firm is fixed. There is no substitute presently available for Miragood technology. Market conditions are also assumed to remain constant.

Worker

Worker, as an entity, is a member of the Firm and of Society. In our model, Worker earns its livelihood solely by offering its labor in exchange for wages. Worker has a vested interest in higher wages. In addition to seeking higher wages, Worker wants a healthy and safe workplace. In the long run, the wage rate of Worker is determined by the supply—demand mechanisms of the labor market within Society, but since the model simulates the short run, the wages rate is assumed to be fixed. Because the relationship between the Firm and Worker is symbiotic, and because of the fixed wage rate assumption above, the short term well being and security of Worker ultimately depend on the continued operation of the Firm. Worker can in the long run switch jobs, but only with the incursion of certain transaction costs, such as the cost of relocation and retraining. In the model the wage rate paid to the worker is dependent on the exposure level of NOX: this is the hazard wage differential. Worker does not bargain with the Firm as a unit and mobility of labor is low. Although the assumption of fixed wage rate and short run immobility is used to develop the central idea, the ramifications of the results under less restrictive, more realistic conditions will be discussed later.

Consumer

Consumer, as an entity, is interested in maximizing its standard of living. Miragood plays a large part in the life of Consumer and Consumer is willing to pay the market price to the Firm in exchange for Miragood. The market price is in part set by Consumer aggregate demand as manifested in the demand curve.

Society

Society is the aggregate will, and sometimes conscience of the Firm, Worker, and Consumer combined. Normally Society adheres to the economic doctrine of laissez-faire, but it ventures away from noninterference under certain circumstances. For instance, it acts as an impartial third party to settle conflicting interests. It also undertakes responsibility for the common defense, polices the public safety and health, and conducts other public works that otherwise are not provided for under the free market.

As the paramount entity, Society is able to take the broader, longer term, and more objective viewpoint in its deliberations, but it too faces practical political constraints when attempting to implement its programs.

The problem

In order to produce Miragood, the Firm uses the chemical NOX as raw material. NOX is a chemical irritant that, in sufficiently high levels, is hazardous to exposed persons. The average NOX concentration in the plant without control technology has been C_{F^0} milligrams per cubic meter. A number of workers exposed to this concentration in the plant suffer from irritation of eyes, skin and the respiratory tract. On an ongoing basis, a fraction of Worker has been forced to stay away from work and seek medical attention for recovery. NOX is a problem to the Firm because the chemical irritant has continually affected a portion of Worker, causing disruption in the Firm's production effort. Fortunately, the effects of NOX are temporary and nonfatal — recovery is complete upon removal and treatment. The affected workers return to work after a recuperation period.

The Firm nevertheless is confronted with work disruption whenever a worker is absent, and it is costly to find a temporary replacement. The Firm does not have to pay wages to an absent worker. Because of the laws and labor practices within Society, the Firm is not required to pay for an affected worker's medical expense. However, there are some other intangible costs to the Firm: morale and worker—management relations are at a low point, and the Firm suspects that overall work efficiency in the plant is being reduced. However, the Firm does not have a solution to the problem of high NOX concentrations in its plant.

Society is aware of NOX through the public media and is concerned over the problem. Ongoing research at Public University on the epidemiology and toxicology of NOX confirms the health effect of NOX to be acute but temporary — average recovery time from an incapacitating exposure of NOX is two weeks with medical attention. Epidemiological data collected over the years give a good indication of the relationship between the number of workers incapacitated and average NOX concentrations in the plant. Figure 1 gives the annual dose—response relationship (the probability of incapacitation on a yearly basis as a function of C, average NOX concentration in milligrams per cubic meter). This conveniently simple relationship can be expressed mathematically as a general probability function

$$P(C) = (C/a)^n - z/a \quad \text{for } C > z \tag{1}$$

and

$$P(C) = 0 \quad \text{for } C < z \tag{2}$$

where $1/a$ is the slope of the fitted curve when $n = 1$. Thus, $(a + z)$ can be interpreted as the concentration that causes an annual 100% incapacitation of the exposed worker population. When $n = 1$, the threshold limit value

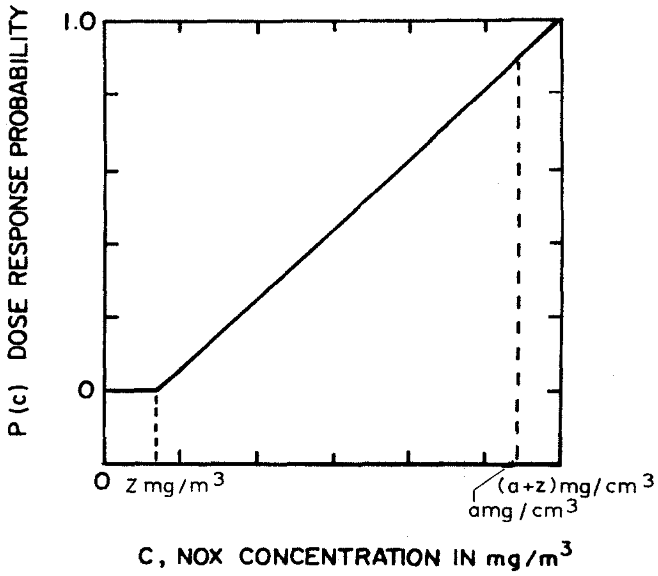


Fig. 1. The dose—response curve.

(i.e., the NOX concentration, C, when $P(C)$ is zero) or TLV is equal to z. However, as we shall see later, the TLV is not equal to z when n is not unity. z is the threshold limit below which no response is observed in the subject. The study also confirmed average concentrations in the plant to be C_{F^0} milligrams per cubic meter. The chronic toxicology of NOX is not well documented.

The control technology solution

Fortunately ventilation engineers at Public University, have come up with a solution. They have successfully tested a ventilation system (Fig. 2) that can reliably reduce NOX concentration in the plant. The NOX concentration depends on the size of the equipment installed. The NOX introduced into the outside environment by the ventilation system is presumed to be diluted into a harmless concentration and thus no harm or cost is imposed on the environment. Upon this announcement, all entities are initially elated, but only for a short period, during which each affected entity hastens to determine for itself the size of the system it desires and the concentration of NOX it can tolerate.

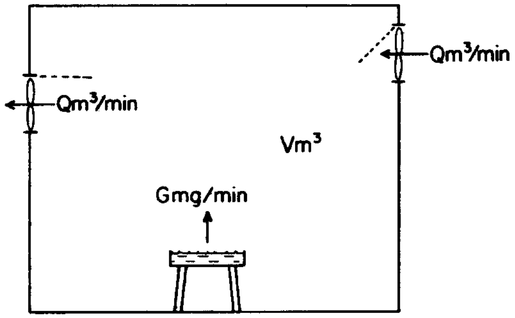


Fig. 2. The ventilation system.

The engineering model

The engineering solution calls for the installation of a ventilation system within the plant. A steady state general dilution ventilation model is used for arithmetic simplicity.

The condition in the plant can be described by a material balance for NOX. Assuming no NOX in the supply air, rate of accumulation of NOX = rate of generation − rate of removal:

$$V\,\mathrm{d}C = G\,\mathrm{d}t - Q'C\,\mathrm{d}t \tag{3}$$

and

$$Q' = Q/K \tag{4}$$

where C = concentration of NOX at time t (mg/m^3); G = rate of emission of NOX from process into plant (mg/min); K = design constant, allowing for incomplete mixing; Q = design rate of ventilation (Q is greater than Q') (m^3/min); Q' = effective rate of ventilation, corrected for incomplete mixing (m^3/min); t = time (min); and V = volume of plant (m^3).

When installed, the system operates at steady state, NOX concentration is kept at a constant level, and

$$\mathrm{d}C = 0 \tag{5}$$

Equation (3) simplifies to

$$G\,\mathrm{d}t = Q'C\,\mathrm{d}t \tag{6}$$

With rate of emission being independent of time, eqn. (6) can be integrated between a time period bounded by t_1 and t_2

$$G\int_{t_1}^{t_2}\mathrm{d}t = Q'C\int_{t_1}^{t_2}\mathrm{d}t \tag{7}$$

$$G(t_2-t_1) = Q'C(t_2-t_1) \tag{8}$$

$$Q' = G/C \tag{9}$$

$$Q = KG/C \tag{10}$$

Equation (7) gives the flowrate required to attain a desired NOX concentration. The volume, V, has dropped out of the steady state solution. When Q is known, the equipment can be selected. The ventilation equipment can be installed commercially according to the cost function, U_e, given below:

$$U_e = AQ^b \tag{11}$$

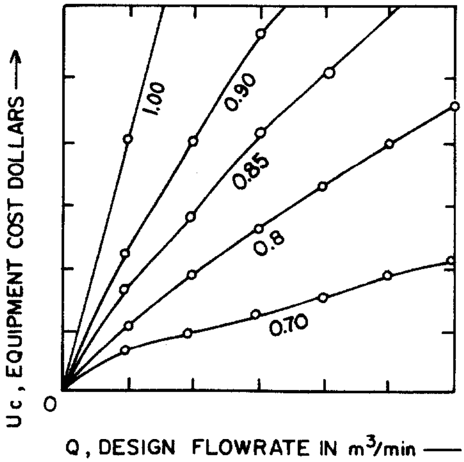


Fig. 3. Cost function with various scale exponents.

where A = the unit installed cost of the equipment for cubic meter per minute of airflow ($\$/m^3$); and b = an economy of scale exponent for the system; b is less than 1. The function is presented graphically in Fig. 3.

The operating cost of the system, U_v, is given by

$$U_v = BQ^b \tag{12}$$

where B = annual operating cost for each cubic meter per minute of airflow; and b = economy of scale factor for operating costs. (Both cost equations use the same scale factor for illustrative/analytical convenience. With the same scale factor, the resulting equations can be solved analytically; otherwise a numerical approximation solution would have to be used.)

Costs can be now expressed in terms of NOX concentration by substituting eqn. (10) into eqns. (11) and (12):

$$U_e = A(KG/C)^b \tag{13}$$

and

$$U_v = B(KG/C)^b \tag{14}$$

Public University has dedicated all engineering information to the public domain. This information was obtained at a certain cost, but it is treated as a sunk cost for Society, just as the capital cost of equipment is a sunk cost for the Firm.

The position argument of the Firm

Firm sought to minimize costs. Since Miragood technology and all other variables are unchanged in the short run, profit is maximized by identifying and minimizing all the costs arising out of solving the NOX problem. Total costs to the Firm on an annual basis, U_F, were found to be

$$U_F = U_e/L + U_v + P(C)NW_F \tag{15}$$

where $L = f(i,y)$, factor for annualizing costs depending on y, the equipment life in years, and i, the discount rate (year^{-1}); N = number of workers in the Firm (persons/year); W_F = total costs per worker sustained by the Firm from a NOX-related absence ($/person); and C_F = the concentration of NOX in the plant with controls.

Since the Firm practices wage differentiation to induce Worker to remain under hazardous jobs, the hazard-sensitive wage rate paid to Worker is a function of the NOX concentration

$$W_F = f[P(C_F)] \tag{16}$$

For the case of perfect wage-hazard differentiation,

$$W_F = y_F W_1[j[(C_F/a)^n - z/a] + 1] \tag{17}$$

where j = a differential constant given as a fraction; it represents the ceiling of the maximum differential as compared to the base wage rate, W_1. For example, $j = 0.5$ means a maximum differential ceiling of 50%; W_1 = the hourly base wage rate without correction of wage differential (\$/h); and y_F = the factor that when multiplied by W_1 gives the total disruptive cost to the Firm from an absence (h/person).

The hourly wage rate is used because it reflects the skill level of the job, and thus is an indication of the ease of replacement and quanta of work disruption from an NOX related absence. This definition is possible because prevailing laws and labor practices do not require the employer to pay wages to a worker who is forced by work-related conditions to be absent. Also, the employer does not pay for any medical expenses incurred by a worker.

By substituting eqns. (1) and (17) into eqn. (15), the cost to the Firm, U_F, can now be expressed as a function of C_F, the NOX concentration

$$\begin{aligned} U_F &= A(KG/C_F)^b/L + B(KG/C_F)^b + y_F NW_1[j(C_F/a)^{2n} \\ &\quad - 2j(zC_F^n/a^{n+1}) + (C_F/a)^n + jz^2/a^2 - z/a] \qquad (18) \end{aligned}$$

This equation cannot be optimized analytically, but the special case of $j = 0$ as given in the next equation can be solved:

$$U_F = [A(KG/C_F)^b]/L + B(KG/C_F)^b + y_F NW_1[(C_F/a)^n - z/a] \qquad (19)$$

Equation (19) is plotted in Fig. 4.

Equation (18) can be solved numerically, and the results simulated by computer. A minimum for eqn. (19) occurs at C_{F*}. This minimum can be derived analytically by differentiating eqn. (19) with respect to C_F, setting the derivative to zero, and solving for C_F.

$$C_{F*} = \left[\frac{a^n b(A/L + B)(KG)^b}{y_F nNW_1}\right]^{1/(b+n)} \qquad (20)$$

This point can be confirmed to be a minimum by taking the second derivative and finding it to be negative for specific values of the equation parameters.

$$\begin{aligned} \mathrm{d}^2U_F/\mathrm{d}C_F^2 &= [b(b+1)(A/L + B)(KG)^b]/C_F(b+2) \\ &\quad + y_F n(n-1)[C_F(n-2)/a^n]NW_1 \qquad (21) \end{aligned}$$

By substituting C_{F*} into C in eqn. (17) the optimal cost for the Firm, U_{F*}, can be found. Management compares this U_{F*} with current costs arising from work absences, U_{F°,

$$U_{F^\circ} = y_F NW_1[(C_{F^\circ}/a)^n - z/a] \qquad (22)$$

If U_{F*} is equal to or less than U_{F°, control technology will be installed to reduce the NOX concentration from C_{F° to C_{F*}. Nothing will be done if U_{F*} is greater than U_{F°. Average C_{F° is available from plant records.

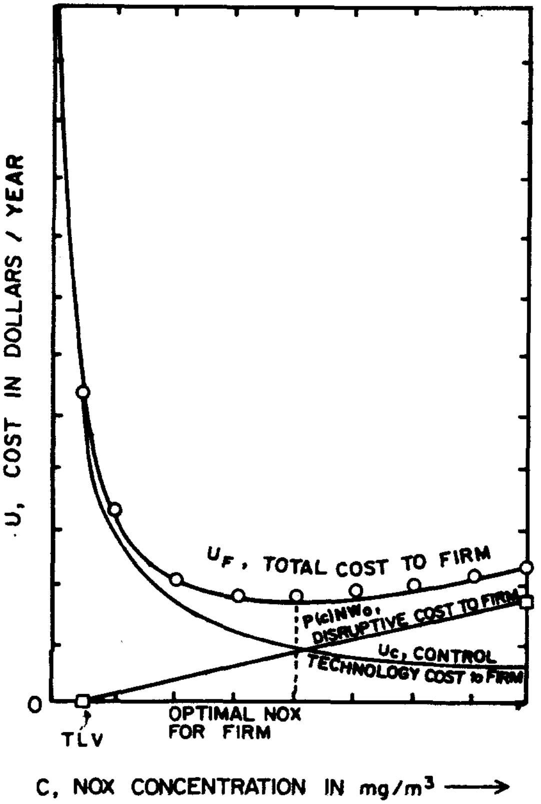


Fig. 4. The cost minimizing NOX concentration of the Firm.

The position of Worker

Worker would intuitively respond with a demand for maximum protection, that is, zero NOX concentration. The irritating effects of NOX are readily recognized, unlike the more latent, but no less detrimental effects, of other chemicals. From eqn. (10), for a NOX concentration of zero,

$$Q = KG/0 = \text{infinity} \tag{23}$$

Consequently the control technology cost to the Firm becomes infinite. The Firm can achieve zero NOX concentration, that is to reduce the emission rate, G, to zero (Q approaches a constant as G and C approach zero simultaneously). In fact reduction of G may be the only alternative if NOX is a hazard to the environment as well as the workplace. This can conceivably be done by designing an emission-proof production system or by substituting NOX with a nonirritant. In the short run, both alternatives are impractical and the Firm may shut down. Thus, upon reflection, Worker can see that it has a lot to lose if excessive mandatory controls were ordered, forcing the Firm to shut down. Worker is impaled on the horns of a dilemma. More importantly, Worker lacks mobility in the labor market, and since it also lacks the bargaining unity afforded, for example, by a union, Worker as an entity is not in a good position to influence decisions. There may also exist as a result of insufficient information, a propensity for Worker to trade off health improvements for an increase in wages.

Consumer's viewpoint

Some members of Consumer would like to see improved working conditions through the rigorous implementation of control technology. On the other hand, in a period of inflation, Consumer values stable prices, especially affordable Miragood prices. It is rumored that the control technology costs imposed upon the Firm will be passed onto Consumer in terms of higher price. Consumer perceives of no immediate replacement for Miragood (in other words, Miragood enjoys a relatively, but not perfectly, inelastic demand curve in the short run), and is worried about having to pay a higher price for Miragood. In addition, the adverse effects of continued production are confined within the limits of the plant, and do not disturb the average consumer with the force of some other more ubiquitous externalities, like, for instance, the harm exacted by environmental pollution, or the concern caused by nuclear power issues. However, some members are concerned about the effect of the NOX that is introduced into the environment by the ventilation system.

Society's analysis and policy

Society, because it represents the aggregate interests of its members, takes a broader approach when evaluating a societal problem. It searches for that elusive common ground for the resolution of differences. It is only logical for Society to want nothing less than the optimal distribution of its limited resources in a way that will maximize the return to the whole (i.e., the Pareto optimal). Nevertheless it is subject to a number of real constraints including that of the insufficiency of information. Under certain circumstances, the market fails to act as a viable pricing mechanism in which resources can be optimally allocated for the maximum return to the whole. To the economists within Society, negative externalities give rise to market failure. The NOX problem is an example of a negative externality. This statement is demonstrated through the mathematical model presented in the pages that follow.

For Society total costs, on an annual basis, are given by

$$U_S = U_F + U_W + U_C \tag{24}$$

where U_W = total costs sustained by Worker annually (\$/yr); and U_C = total costs sustained by Consumer annually (\$/yr). Specifically,

$$U_W = N(y_W W_1 + W_2)[(C_F/a)^n - z/a] + NW_3(C_F/a)^n \tag{25}$$

where y_W = the average number of work hours lost per worker (h/person); W_1 = hourly wage rate; this multiplied by y_W gives the total wage lost per absence (\$/h); W_2 = average medical expense for each affected worker; medical expenses include treatment for psychological effects, if any (\$/person); and W_3 = the comfort constant (\$).

Note that there is no TLV in the cost of comfort expression. Assuming the Firm will absorb the control technology costs, and that the NOX removed from the plant into the general environment will be diluted to a harmless level and not impose a cost on Consumer

$$U_C = 0 \tag{26}$$

If the environment is harmed by NOX, then U_C will take on a positive value. In such an instance, the ventilation technology is rendered inappropriate and should not be used as it merely shifts the harm from one area into another, unless an effective separation process (e.g., a scrubber, electrostatic separator or baghouse) is available. In this case the only way to reduce total cost is to reduce G. At this point, note that the model is optimizing for the control technology level, that is the optimal allocation of Society's resources by identifying all the harm caused by NOX. It is not answering the question of on whom should the costs fall. The assignment of a positive function to eqn. (24) while simultaneously assigning cost to the Firm via eqn. (15) would amount to double counting.

The preceding discussion implies that under the free market each entity tends to exhibit short-sightedness, thereby causing an overall undercounting of costs. A definition for an externality can now be introduced: an externality occurs whenever entity A selects its utility or production function without due regard for entity B's utility or production function because entity A's cost allocation system does not account for adverse impacts on entity B. In other words, because of the way costs are defined, the costs that are actually borne by entity B are external to entity A, and for the purposes of entity A, simply do not exist. The cost burden of the externality is shifted wholesale, with an outright circumvention of the marketplace, without the benefit of offer and acceptance, onto entity B. No value or price can be placed on the externality by the market because, under existing conditions, no market for pricing the negative value of NOX exists. In the model, the labor or legal market is incapable of resolving the problem. There has been a market failure with corresponding malfunction of the pricing system. New technology may on the whole generate net benefits, but in the producer's haste to reap the technological advantage, and because of uncritical acceptance of the more apparent usefulness of the technology by the consumer, both producer and user have often in effect ignored or overlooked the inherent costs of technology, and thus have shifted the burden on another unsuspecting or captive entity. The acceptance may be hastened by aggressive marketing. The existing laws and custom may be conducive to such oversight.

Hence, the first task is to identify and fully count the costs, and afterwards be concerned with the allocation of the costs. In all probability, the marketplace of old may be able to sort out the pricing readjustment needed. Substituting eqns. (19) and (25) into eqn. (24),

$$\begin{aligned} U_S &= (A/L + B)(KG/C_F)^b + y_F N W_1 [(C_F/a)^n - z/a] \\ &\quad + N(y_W W_1 + W_2)[(C_F/a)^n - z/a] \\ &\quad + N W_3 (C_F/a)^n \end{aligned} \tag{27}$$

Equation (25) for the case when $W_3 = 0$ is shown graphically in Fig. 5.

The cost to Society without any control technology, U_{S° is

$$U_{S^\circ} = N[(y_F + y_W)W_1 + W_2][(C_{F^\circ}/a)^n - z/a] + N W_3 (C_{F^\circ}/a)^n \tag{28}$$

The same analytical technique is used to find the C_{S*}, the optimal NOX concentration for Society

$$C_{S*} = \left[\frac{a^n b (A/L + B)(KG)^b}{nN[(y_F + y_W)W_1 + W_2 + W_3]} \right]^{1/(b+n)} \tag{29}$$

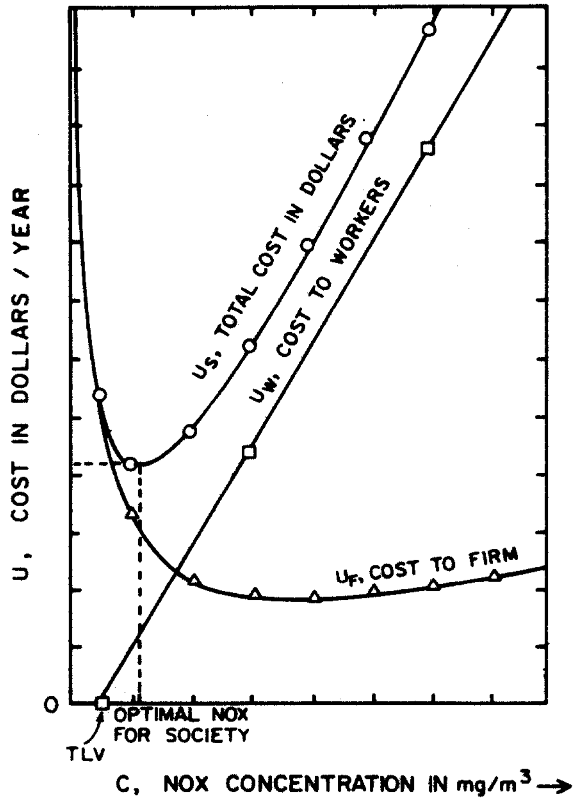


Fig. 5. The optimal NOX concentration for Society.

A comparison between eqns. (29) and (20) shows that U_{F*} is necessarily smaller than U_{S*} because the equations are the same except for the positive terms in the denominator. The equations therefore demonstrate that the Firm (and Worker who for a wage becomes a part of the Firm and henceforth becomes exposed to NOX) is willing to tolerate a higher level of NOX exposure than the level which is optimal for Society. A graphical comparison of the two equations can be made by comparing Figs. 4 and 5. The optimal concentration of NOX is not zero because Consumer as an entity, through the market pricing mechanism, has chosen not to forego the economic rent (i.e., benefit) derived from the use of Miragood, and therefore 'the net benefits derived from the use of Miragood can be, for the time being and without redefining the assumptions used so far, presumed to exceed the inherent costs and risks. However, the optimal concentration as calculated by Society should be the concentration allowed, as it minimizes costs and therefore maximizes benefits, *ceteris paribus*, for the whole. If the Firm was required to adopt the Society optimal, it is very likely that the Firm will attempt to shift at least a portion of the cost to the Consumer by raising Miragood prices. When this more complete cost is integrated into the Miragood market and pricing system, the demand for Miragood will decrease, and the elasticity of demand will determine the business success of Miragood in the marketplace.

The benefit and the decision index

The benefit (BN) is defined simply as the difference between the cost to an entity without the control technology and the cost to the entity with the control technology. For Society and the Worker, $\mathrm{BN_S}$,

$$\mathrm{BN_S} = U_{S^\circ} - U_S \tag{30}$$

and for the Firm, $\mathrm{BN_F}$,

$$\mathrm{BN_F} = U_{F^\circ} - U_F \tag{31}$$

A negative benefit means a misallocation of resources if the control technology is installed. The decision index ratios, R_F and R_S, are the quotients obtained by dividing the dollar amount of benefit by the dollar amount of the cost incurred by the Firm and by Society upon installation of the control technology:

$$R_S = \mathrm{BN_S}/U_S \tag{32}$$

and

$$R_F = \mathrm{BN_F}/U_F \tag{33}$$

A numerical example

The following values are assigned to the parameters:

(a) $a = 200$ mg/m^3
(b) A = \$800 per m^3/min of airflow
(c) $b = 0.8$, the economy of scale factor
(d) B = \$25 per m^3/min of airflow annually
(e) $C_{F^0} = 80$ mg/m^3
(f) $G = 5000$ mg/min (300 g/h) of NOX emitted into plant
(g) $K = 5$, the correction factor for mixing
(h) $L = 8.5136$, the annualizing factor when i, the discount rate, is 10% and y, the equipment life, is 20 years
(i) $n = 1$
(j) W_2 = \$750 average for each affected worker (\$50 per day for average absence of two weeks)
(k) $W_3 = 0$ because it is too much of an intangible to be subject to accurate quantification
(l) $y_F = 24$ h
(m) $y_W = 80$ h
(n) $z = 5$ mg/m^3, the response threshold limit value

The above values will be held constant throughout the analysis. The other parameters, N and W_1, will be varied to illustrate the effect of their changes on the total costs, U_{F*} and U_{S*}. The number of workers, N, will be given values of 100, 500 and 2000 persons.

The wage rate, W_1, is given values of \$4.00 per hour and \$12.00 per hour. The NOX concentration that minimizes total cost is calculated for various combinations of N and W_1 from eqns. (20) and (29). Equation (10) gives the Q design. Total control technology cost to the Firm, U_F, is found from eqn. (19) and that of Society, from eqn. (27). Cost without the control technology, as given by eqns. (22) and (28), respectively, are presented alongside for comparison. The solution according to the response threshold limit value (TLV) criteria is:

TLV = 5 mg/m^3, this being the concentration that causes no disruptive harmful effects (but, unfortunately, not necessarily no discomfort)
Q_{TLV} = 5,000 m^3/min
U_{TLV} = \$108,294 per year (total cost as limited by the TLV).

This cost is the same for both Society and the Firm. Any reduction of NOX concentration below the TLV of 5 mg/m^3 will yield no additional monetary return to both Society and the Firm. Such a reduction may result in greater working comfort, but this particular example does not assign monetary value to worker comfort. The assignment of monetary value is basically an axiological decision. If the reader prefers to use different numbers for the various parameters, the model will yield a different set of answers that are appropriate to the values used. The model contains general parameters that can cover a broad range of value judgements. Its usefulness rests in its generality. Like other cost—benefit models, there is difficulty in measuring social cost and benefit. Nevertheless, for occupational health, the system is smaller and better defined than that of the environment. The numerical results are presented below:

Case I: W_1 = \$4.00 per hour; N = 100 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	132	190	14,000	3,600	−10,400	−0.74	Do not install
Society:	33	760	40,300	43,700	3,500	0.09	Install

Case II: W_1 = \$4.00 per hour; N = 500 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	54	460	27,900	18,000	−9,900	−0.35	Do not install
Society:	13	1,860	73,700	218,600	144,900	1.97	Install

Case III: W_1 = \$4.00 per hour; N = 2000 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	25	1,000	49,100	72,000	22,900	0.47	Install
Society:	6	4,010	105,200	874,500	769,300	7.32	Install

Case IV: W_1 = \$12.00 per hour; N = 100 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	72	350	22,500	10,800	−11,700	−0.52	Do not install
Society:	24	1,020	49,900	74,900	25,100	0.50	Install

Case V: W_1 = \$12.00 per hour; N = 500 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	29	850	43,800	54,000	10,200	0.23	Install
Society:	10	2,500	87,200	374,600	287,500	3.30	Install

Case VI: W_1 = \$12.00 per hour; N = 2000 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	14	1,850	73,400	216,000	142,600	1.94	Install
Society:	4.6**	5,410	107,800**	1,498,500	1,390,700	12.90	Install to TLV

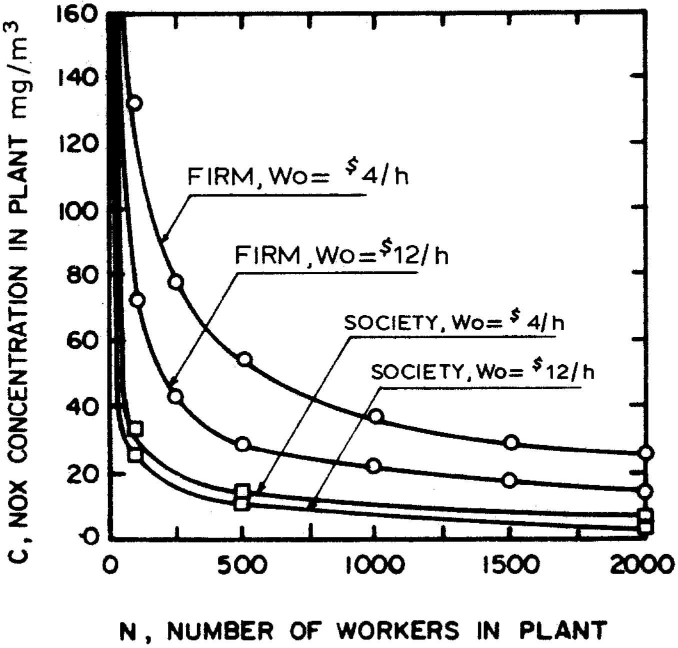


Fig. 6. A comparison of the optimal NOX concentration for Society and the cost minimizing NOX concentration for the Firm for fixed emission rate.

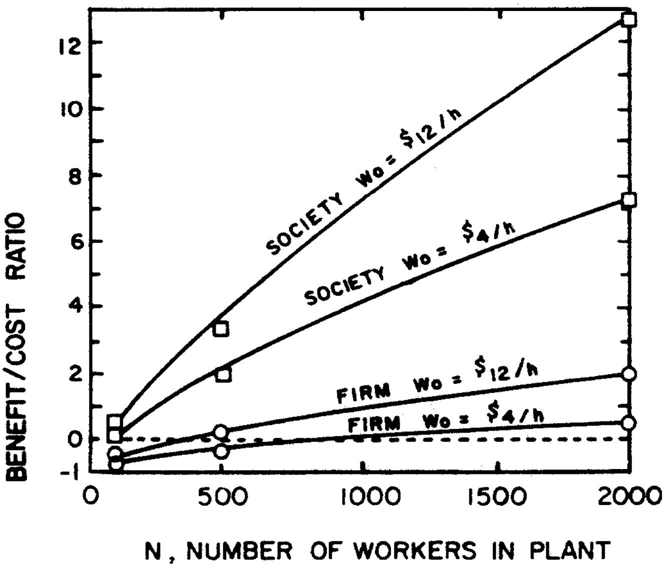


Fig. 7. A comparison of decision index ratios for fixed emission rate.

The double asterisk in Case VI indicates a C Optimal for Society that exceeds the TLV of 5 mg/m^3 and hence is an overdesign. The optimum is actually at the TLV of 5 mg/m^3 and a cost of $108,295.

The numerical results are presented graphically in Figs. 6 and 7.

The case of varying rate of emission

The next numerical example uses a rate of emission that is a function of the number of workers in the Firm:

$$G = \mathrm{f}(N) \tag{34}$$

Specifically,

$$G = gN + g_k \tag{35}$$

where g = rate constant of NOX emitted for each worker (mg/min person); and g_k = residual rate of emission of NOX in the plant.

For illustrative purposes, let g = 5 mg/min person, and g_k = 1000 mg/min. The results are presented below.

Case VII: W_1 = $4.00 per hour; N = 100 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost ($)	Cost w/o CT ($)	BN ($)	B/C ratio	*Decision*
The Firm:	77	97	8,100	3,600	−4,500	−0.56	Do not install
Society:	19	390	22,400	43,700	21,300	0.95	Install

Case VIII: W_1 = $4.00 per hour; N = 500 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost ($)	Cost w/o CT ($)	BN ($)	B/C ratio	*Decision*
The Firm:	46	380	23,600	18,000	−5,600	−0.24	Do not install
Society:	11	1,520	60,800	218,600	157,800	2.60	Install

Case IX: W_1 = $4.00 per hour; N = 2000 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost ($)	Cost w/o CT ($)	BN ($)	B/C ratio	*Decision*
The Firm:	35	1,550	71,700	72,000	300	0.00	Install
Society:	9	6,220	173,800	874,500	700,700	4.03	Install

Case X: W_1 = $12.00 per hour; N = 100 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost ($)	Cost w/o CT ($)	BN ($)	B/C ratio	*Decision*
The Firm:	42	180	12,900	10,800	−2,100	−0.16	Do not install
Society:	14	520	44,400	74,900	30,500	0.69	Install

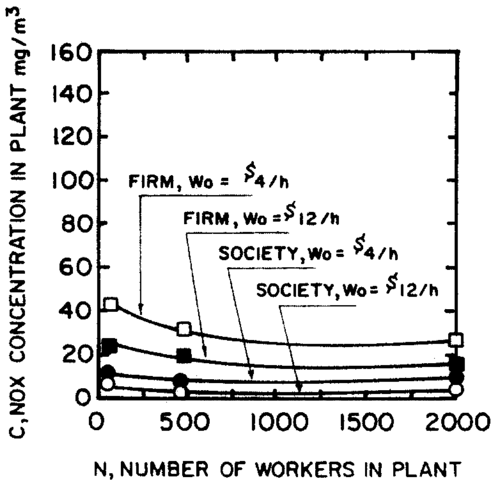
Case XI: W_1 = \$12.00 per hour; N = 500 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	25	700	36,900	54,000	17,100	0.46	Install
Society:	9	2,050	70,700	374,600	303,900	4.30	Install

Case XII: W_1 = \$12.00 per hour; N = 2000 workers.

	C Optimal (mg/m^3)	Q Design (m^3/min)	Total cost (\$)	Cost w/o CT (\$)	BN (\$)	B/C ratio	*Decision*
The Firm:	19	2,860	110,300	216,000	105,700	0.96	Install
Society:	7	8,390	194,900	1,498,500	1,303,600	6.69	Install to TLV

The numerical results are plotted in Figs. 8 and 9 (cf. Figs. 6 and 7).



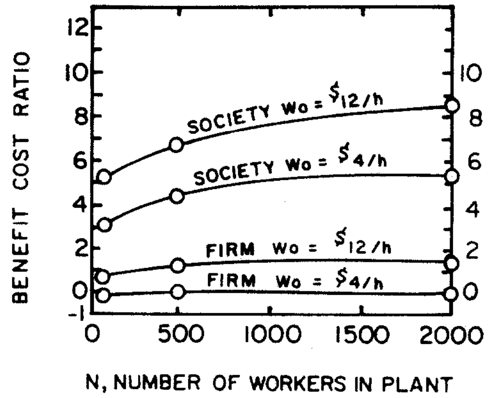


Fig. 8. A comparison of the optimal NOX concentration for Society and the cost minimizing NOX concentration for the Firm for a variable emission rate.

Fig. 9. A comparison of decision index ratios for a variable emission rate.

A brief discussion on the ramifications of control technology

A discussion of the ramifications will necessarily involve an analysis of the long run. If C_{S*}, the optimal NOX concentration for Society, is selected as the policy level, then the Firm will have to bear the cost of retrofit control technology initially, provided that its present control technology is below Society expectations. However, it is unlikely that the Firm will absorb this additional cost in the long run, since it is more probable that the Firm will attempt to pass the cost to Consumer by raising the price of Miragood. The ability of the Firm to pass the cost will depend on the elasticity of demand for Miragood. Although the model was developed for a monopolistic producer, the assumption of monopoly can be lifted without excessive adversity, because under perfect competition, the producers' supply curve is the aggregate total of the individual producers. Under perfect competition, the individual producer cannot affect the price of its product, and it either operates at a profit, or it will shutdown to avoid a loss, both outcomes being dependent on the demand curve. Even in a monopoly where the single producer can affect the price of its product, the possibility of a shutdown exists because the control technology policy affects the fundamental cost of production. The producer cannot avoid or change this basic additional cost. Of course, the impact of this additional cost will be felt sooner in the marketplace with a monopoly than under perfect competition or imperfect competition (e.g., a duopoly or oligopoly). A greater time lag characterizes the adjustment process of imperfect competitors as they each try to probe and react to the pricing strategies of the others. However, as a matter of equity, the enforcement of Society's policy must be applied without exception to all producers if there are more than one. However, because the model was derived by the microscopic examination of a single producer, it is sensitive to the size of the producer as measured by the number of workers, and will adjust for the smaller producer by requiring a corresponding lower level of control technology. Nevertheless, the unit technology cost ($\$/m^3$) to each producer will vary because of the intrinsic economy of scale involved, and also there exists in real life, different levels of technical competence amongst producers in the absence of technology sharing. Thus the overall competitiveness of each producer may be enhanced or diminished depending upon a host of factors.

In the final analysis, however, it appears that Consumer retains the last word on the future of Miragood — if a substitute replaces Miragood after the increase in price, then it is clear that Miragood is no longer an economically and socially desirable product in the long run, and it will eventually decrease in market importance. In such an event, the scarce resources of Society will be diverted more beneficially to the production of other goods. Of course the demand curve enjoyed by Miragood may dictate otherwise. Besides the use of a hazard wage differential, the effects of collective bargaining and other refinements pertaining to the labor—firm interactions are not accounted for in the model's development because the model analyzes events under short run conditions. Additionally, the complex interactions between the supplier and consumer of labor do not lend themselves easily to mathematical modeling. Nevertheless if for example, the wage differential model is solved numerically (i.e., setting j to be greater than zero) then a more realistic result may ensue.

On the axiological question of death and the price of life

A glaring omission that may have bothered some readers to this point is the absence of discussion on the issue of death from occupational exposure to toxic agents and how the model can account for the value of human life. This is indeed a most difficult axiological question which ventures deep into the realm of philosophers and medical ethicians. Nevertheless, a discussion will be attempted here as the problem, while on the one hand metaphysical, is also very real and urgent.

First, there are at least two situations under which the valuation of human life is made: (1) before death when the worker is alive, and (2) after death. The second situation is easier to rationalize and accept if the worker is indeed dead from an occupational disease or injury, for then he is dead and the only thing left that we can do is to calculate the equitable compensation to his or her survivors. This is done regularly in worker compensation cases and in the determination of damages in a wrongful death trial. In a sense the true question is rendered irrevocably moot by the death which created the question in the first place, for nothing within our current knowledge will reverse the process of death. The difficult and living question is what happens afterwards? What is society going to do to prevent a recurrence of the disease or injury that killed before? An even more perplexing problem is what is society going to do to avert a possible death from an agent that has the potential to kill, a potential that has been demonstrated on test animals in the Laboratory? How much is society going to spend to protect the living worker from death in the workplace? Do we have to put a price on life in order to determine the answer?

This is an axiological question that the model cannot answer alone. The cost of imminent death to any individual faced with death is quite certainly very high, even infinite. If society accepts this individual cost of infinity and we account for this in the model by setting W_3 to infinity, then eqn. (29) gives a C_{S*} of zero. Unless there is a breakthrough in control technology that renders eqns. (13) and (14) invalid, then the cost of control will go to infinity. An infinite cost is simply beyond the means of any society for we face a situation of limited resources and scarcity.

However, the public policy maker can use a very large W_3 and then eqn. (29) will yield a very small but non-zero number for concentration. Thus the model will assist the decision-maker to arrive at a rational quantitative answer, but only after the decision-maker has first supplied the necessary axiological input. It is most unfortunate and regretable that the model breaks down mathematically when it encounters either zero or infinity in certain of its parameters.

Of course Society may completely forego the use of Miragood and thus not worry about NOX anymore.

Conclusions

The 3-E model quantitatively shows the following:

(1) The model explicitly minimizes the total cost of all the entities. Thus, the model represents a compromise from the goals of each individual entity but, nevertheless, is the best for the whole. The need for a rational and balanced search for society's optimal takes on additional urgency in a period of shrinking resources and rising prices.

(2) The Firm, with the cost accounting system as defined and a profit maximizing (cost minimization) criterion will opt for a higher concentration of NOX in the work environment than will Society. The Firm probably will not operate totally without any control technology because of the cost of work disruption. However, it will also not install any control technology unless the Firm's cost minimization analysis justifies the cost of installation. Society's threshold for the installation of control technology is lower than that of the Firm's because Society recognizes more costs which in fact exist than does the Firm. The actual limiting concentration desired is dependent on the types and value of social costs recognized and counted within Society.

(3) The economic theory of externalities is applicable. However, a number of distinctions exist between the occupational health externality and the other common externality of environmental pollution. For instance, the pollution of the environment impacts ubiquitous public goods like clean air and water, both of which are needed by all consumers, while the occupational health externality is confined to workers within the plant. In addition to this insulation effect, an inherent dichotomy exists between lower consumer product prices and improved health for the worker. The above, however, is counterbalanced by the fact that industrial productivity is directly affected by worker safety and health, worker morale in general, and individual productivity. The nexus between industrial vitality and environmental well-being is less apparent on its face. Because of the recognition of more costs by Society (costs that are external to or nonexistent in the business accounting system) Society derives a higher decision index ratio than the Firm (see Fig. 7). Risk sharing efforts implemented from a broader level are necessary to overcome occupational health problems. Government regulation is necessary in cases of market failure but the regulations must account for all costs and seek to achieve optimal allocation of Society's resources.

(4) The optimal NOX level for Society and the profit maximization NOX level for the Firm will decrease when wages of Worker increase. This will happen because as the wages, and therefore sophistication of labor skills increase, both Society and the Firm can afford a lesser amount of health and work disruption, respectively.

(5) The optimal NOX level for Society and the cost minimizing (or profit maximizing) NOX level of the Firm will decrease when the number of workers in the plant increases. This is observed in the field: smaller businesses with less employees have less incentive and greater difficulty matching the

control technology programs available in larger companies with more employees. A larger number of employees allows the employer to exploit the economy of scale of the control technology. It is also true that companies with more employees usually are in a stronger financial position to afford the control technology.

(6) As can be perceived, the model is a technique that can be used as an analytical tool for studying the risks, costs, and the maximizing of benefits from the use of control technology in the industrial environment. The results will depend on the numerical values used in the equation parameters. A variety of model results are conceivable, and the challenge may rest in the actual determination of the numerical values of the parameters to be inserted into the model equations outlined above. However, as the system boundary for the industrial health environment is smaller and better defined than the environment at large, the measurement of social costs and benefits should be easier. Nevertheless, the final decision can be an axiological one.

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